Comparaison des courbes terminales

Centres de masse d'ordre deux

Conditions de Keelhoff

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm \quad TOL := 10^{-12}$

Elinvar $\quad \rho_s = 8\times 10^3\ m^{-3}\cdot kg \quad E = 1.7\times 10^{11}\,Pa \quad G = 6.538\times 10^{10}\,Pa$

Parie cylindrique $\quad \psi_0(n_s) := n_s\cdot 360\cdot deg \quad L(n_s) := R_0\cdot\psi_0(n_s)$

$r_s(\alpha) := R_0 \quad s(\alpha) := R_0\cdot\alpha \quad x_{0s}(\alpha) := R_0\cdot cos(\alpha) \quad y_{0s}(\alpha) := R_0\cdot sin(\alpha)$

Courbes terminales en arc de cercle

Courbe terminale externe $\quad \beta := 121\cdot deg \quad \beta_0 := racine\left[\beta\cdot\left(\sqrt{2}\cdot sin(\beta) - 1\right) + sin(\beta)\cdot cos(\beta), \beta\right] \quad \beta_0 = 121.21\,deg$

$$\alpha_A := \pi \quad r_t := \frac{R_0}{\sqrt{2}\cdot sin(\beta_0)} \quad x_{0t}(\alpha_t) := -R_0 + r_t\cdot(1 + cos(\alpha_t)) \quad y_{0t}(\alpha_t) := r_t\cdot sin(\alpha_t) \quad l_t := r_t\cdot 2\cdot\beta_0$$

Courbe terminale interne $\quad \alpha_B(n_s) := mod(\psi_0(n_s) + \pi, 2\cdot\pi)$

$$x_{0t'}(n_s,\alpha_{t'}) := \left[R_0 + r_t\cdot(-1 + cos(\alpha_{t'}))\right]cos(\alpha_B(n_s)) - r_t\cdot sin(\alpha_{t'})\cdot sin(\alpha_B(n_s))$$

$$y_{0t'}(n_s,\alpha_{t'}) := \left[R_0 + r_t\cdot(-1 + cos(\alpha_{t'}))\right]sin(\alpha_B(n_s)) + r_t\cdot sin(\alpha_{t'})\cdot cos(\alpha_B(n_s)) \qquad L_t(n_s) := 2\cdot l_t + L(n_s)$$

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Partie cylindrique $\quad s_s(\alpha) := R_0\cdot(\alpha - \pi) + l_t \qquad z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha)$

$$Z_{2s}(n_s) := \frac{2}{L_t(n_s)^2}\cdot\int_{\pi}^{\psi_0(n_s)+\pi} s_s(\alpha)\cdot z_{0s}(\alpha)\cdot R_0\,d\alpha$$

Courbe terminale externe $\quad \alpha_{tP} := \pi - 2\cdot\beta_0 \qquad z_{0t}(\alpha_t) := x_{0t}(\alpha_t) + i\cdot y_{0t}(\alpha_t)$

$$Z_{2t}(n_s) := \frac{2}{L_t(n_s)^2}\cdot\int_{\alpha_{tP}}^{\pi} r_t\cdot(\alpha_t - \alpha_{tP})\cdot z_{0t}(\alpha_t)\cdot r_t\,d\alpha_t$$

Courbe terminale interne $\quad z_{0t'}(n_s,\alpha_{t'}) := x_{0t'}(n_s,\alpha_{t'}) + i\cdot y_{0t'}(n_s,\alpha_{t'})$

$$Z_{2t'}(n_s) := \frac{2}{L_t(n_s)^2}\cdot\int_{0}^{2\cdot\beta_0} \left(l_t + L(n_s) + r_t\cdot\alpha_{t'}\right)\cdot z_{0t'}(n_s,\alpha_{t'})\cdot r_t\,d\alpha_{t'}$$

Spiral complet $\quad Z_{AC}(n_s) := Z_{2t}(n_s) + Z_{2s}(n_s) + Z_{2t'}(n_s)$

Courbes terminales formées de deux arcs de cercle

Courbe terminale externe

$$r_{t1} := 0.8 \qquad r_{t1} := racine\left[(2\cdot r_{t1}-1)^4 - 4\cdot(1-r_{t1})^4 - \pi^2\cdot r_{t1}^{\,2}\cdot(1-r_{t1})^2, r_{t1}\right]\cdot R_0 \qquad r_{t1} = 0.832\,R_0$$

$$r_{t2} := 2\cdot r_{t1} - R_0 \qquad r_{t2} = 0.665\,R_0 \qquad \beta_0 := arctan\left[\frac{\pi\cdot r_{t1}}{2\cdot(R_0 - r_{t1})}\right] \qquad \beta_0 = 82.695\,deg \qquad l_t := r_{t2}\cdot\beta_0 + \pi\cdot r_{t1}$$

$$x_{0t1}(\alpha_t) := -R_0 + r_{t1}\cdot(1+cos(\alpha_t)) \qquad y_{0t1}(\alpha_t) := r_{t1}\cdot sin(\alpha_t)$$

$$x_{0t2}(\beta_t) := r_{t2}\cdot cos(\beta_t) \qquad y_{0t2}(\beta_t) := r_{t2}\cdot sin(\beta_t) \qquad \alpha_P := -\beta_0 \qquad \alpha_P = -82.695\,deg$$

Courbe terminale interne

$$\alpha_B(n_s) := mod(\psi_0(n_s) + \pi, 2\cdot\pi)$$

$$x_{0t'1}(n_s,\alpha_{t'}) := (R_0 - r_{t1} + r_{t1}\cdot cos(\alpha_{t'}))\cdot cos(\alpha_B(n_s)) - r_{t1}\cdot sin(\alpha_{t'})\cdot sin(\alpha_B(n_s))$$

$$y_{0t'1}(n_s,\alpha_{t'}) := (R_0 - r_{t1} + r_{t1}\cdot cos(\alpha_{t'}))\cdot sin(\alpha_B(n_s)) + r_{t1}\cdot sin(\alpha_{t'})\cdot cos(\alpha_B(n_s))$$

$$x_{0t'2}(n_s,\beta_{t'}) := r_{t2}\cdot cos(\beta_{t'})\cdot cos(\alpha_B(n_s)+\pi) - r_{t2}\cdot sin(\beta_{t'})\cdot sin(\alpha_B(n_s)+\pi)$$

$$y_{0t'2}(n_s,\beta_{t'}) := r_{t2}\cdot cos(\beta_{t'})\cdot sin(\alpha_B(n_s)+\pi) + r_{t2}\cdot sin(\beta_{t'})\cdot cos(\alpha_B(n_s)+\pi) \qquad L_t(n_s) := 2\cdot l_t + L(n_s)$$

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Partie cylindrique

$$s_s(\alpha) := R_0\cdot(\alpha - \pi) + l_t \qquad z_{0s}(\alpha) := x_{0s}(\alpha) + i\cdot y_{0s}(\alpha)$$

$$Z_{2s}(n_s) := \frac{2}{L_t(n_s)^2}\cdot\int_{\pi}^{\psi_0(n_s)+\pi} s_s(\alpha)\cdot z_{0s}(\alpha)\cdot R_0\,d\alpha$$

Courbe terminale externe

$$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + i\cdot y_{0t1}(\alpha_t) \qquad z_{0t2}(\beta_t) := x_{0t2}(\beta_t) + i\cdot y_{0t2}(\beta_t)$$

$$Z_{2t}(n_s) := \frac{2}{L_t(n_s)^2}\cdot\left[\int_0^{\pi}(r_{t1}\cdot\alpha_t + r_{t2}\cdot\beta_0)\cdot z_{0t1}(\alpha_t)\cdot r_{t1}\,d\alpha_t + \int_{-\beta_0}^{0} r_{t2}\cdot(\beta_0+\beta_t)\cdot z_{0t2}(\beta_t)\cdot r_{t2}\,d\beta_t\right]$$

Courbe terminale interne

$$z_{0t'1}(n_s,\alpha_{t'}) := x_{0t'1}(n_s,\alpha_{t'}) + i\cdot y_{0t'1}(n_s,\alpha_{t'})$$

$$z_{0t'2}(n_s,\beta_{t'}) := x_{0t'2}(n_s,\beta_{t'}) + i\cdot y_{0t'2}(n_s,\beta_{t'})$$

$$Z_{2t'1}(n_s) := \frac{2}{L_t(n_s)^2}\cdot\int_0^{\pi}(l_t + L(n_s) + r_{t1}\cdot\alpha_{t'})\cdot z_{0t'1}(n_s,\alpha_{t'})\cdot r_{t1}\,d\alpha_{t'}$$

$$Z_{2t'2}(n_s) := \frac{2}{L_t(n_s)^2}\cdot\int_0^{\beta_0}(l_t + L(n_s) + r_{t1}\cdot\pi + r_{t2}\cdot\beta_{t'})\cdot z_{0t'2}(n_s,\beta_{t'})\cdot r_{t2}\,d\beta_{t'}$$

$$Z_{2t'}(n_s) := Z_{2t'1}(n_s) + Z_{2t'2}(n_s)$$

Spiral complet

$$Z_{2AC}(n_s) := Z_{2t}(n_s) + Z_{2s}(n_s) + Z_{2t'}(n_s)$$

Courbes terminales formées de deux arcs de cercle et d'une droite

Courbe terminale externe

$r_t := 0.5 \cdot R_0 \qquad l_t := R_0 + \pi \cdot r_t \qquad \alpha_A := \pi$

$x_{0t1}(\alpha_t) := r_t \cdot (1 + \cos(\alpha_t)) \qquad y_{0t1}(\alpha_t) := r_t \cdot \sin(\alpha_t) \qquad x_{0t2}(x) := x \qquad y_{0t2}(x) := r_t$

$x_{0t3}(\beta_t) := -r_t \cdot (1 + \sin(\beta_t)) \qquad y_{0t3}(\beta_t) := r_t \cdot \cos(\beta_t)$

Courbe terminale interne

$\alpha_B(n_s) := \operatorname{mod}(\psi_0(n_s) + \pi, 2 \cdot \pi)$

$x_{0t'1}(n_s, \alpha_{t'}) := x_{0t1}(\alpha_{t'}) \cdot \cos(\alpha_B(n_s)) - y_{0t1}(\alpha_{t'}) \cdot \sin(\alpha_B(n_s))$

$y_{0t'1}(n_s, \alpha_{t'}) := x_{0t1}(\alpha_{t'}) \cdot \sin(\alpha_B(n_s)) + y_{0t1}(\alpha_{t'}) \cdot \cos(\alpha_B(n_s))$

$x_{0t'2}(n_s, x) := x_{0t2}(x) \cdot \cos(\alpha_B(n_s)) - y_{0t2}(x) \cdot \sin(\alpha_B(n_s))$

$y_{0t'2}(n_s, x) := x_{0t2}(x) \cdot \sin(\alpha_B(n_s)) + y_{0t2}(x) \cdot \cos(\alpha_B(n_s))$

$x_{0t'3}(n_s, \beta_{t'}) := x_{0t3}(\beta_{t'}) \cdot \cos(\alpha_B(n_s)) - y_{0t3}(\beta_{t'}) \cdot \sin(\alpha_B(n_s))$

$y_{0t'3}(n_s, \beta_{t'}) := x_{0t3}(\beta_{t'}) \cdot \sin(\alpha_B(n_s)) + y_{0t3}(\beta_{t'}) \cdot \cos(\alpha_B(n_s)) \qquad L_t(n_s) := 2 \cdot l_t + L(n_s)$

Centre de masse d'ordre 2

Partie cylindrique

$s_s(\alpha) := R_0 \cdot (\alpha - \pi) + l_t \qquad z_{0s}(\alpha) := x_{0s}(\alpha) + \mathrm{i} \cdot y_{0s}(\alpha)$

$$Z_{2s}(n_s) := \frac{2}{L_t(n_s)^2} \cdot \int_{\pi}^{\psi_0(n_s)+\pi} s_s(\alpha) \cdot z_{0s}(\alpha) \cdot R_0 \, d\alpha$$

Courbe terminale externe

$z_{0t1}(\alpha_t) := x_{0t1}(\alpha_t) + \mathrm{i} \cdot y_{0t1}(\alpha_t) \qquad z_{0t2}(x) := x_{0t2}(x) + \mathrm{i} \cdot y_{0t2}(x) \qquad z_{0t3}(\beta_t) := x_{0t3}(\beta_t) + \mathrm{i} \cdot y_{0t3}(\beta_t)$

$$Z_{2t12}(n_s) := \frac{2}{L_t(n_s)^2} \cdot \left[\int_0^{\frac{\pi}{2}} r_t \cdot \alpha_t \cdot z_{0t1}(\alpha_t) \cdot r_t \, d\alpha_t - \int_{r_t}^{-r_t} \left(r_t \cdot \frac{\pi}{2} + r_t - x \right) \cdot z_{0t2}(x) \, dx \right]$$

$$Z_{2t}(n_s) := Z_{2t12}(n_s) + \frac{2}{L_t(n_s)^2} \cdot \int_0^{\frac{\pi}{2}} \left(r_t \cdot \frac{\pi}{2} + 2 \cdot r_t + r_t \cdot \beta_t \right) \cdot z_{0t3}(\beta_t) \cdot r_t \, d\beta_t$$

Courbe terminale interne

$z_{0t'1}(n_s, \alpha_{t'}) := x_{0t'1}(n_s, \alpha_{t'}) + \mathrm{i} \cdot y_{0t'1}(n_s, \alpha_{t'})$

$z_{0t'2}(n_s, x') := x_{0t'2}(n_s, x') + \mathrm{i} \cdot y_{0t'2}(n_s, x') \qquad z_{0t'3}(n_s, \beta_{t'}) := x_{0t'3}(n_s, \beta_{t'}) + \mathrm{i} \cdot y_{0t'3}(n_s, \beta_{t'})$

$$Z_{2t'1}(n_s) := \frac{2}{L_t(n_s)^2} \cdot \int_0^{\frac{\pi}{2}} \left(l_t + L(n_s) + r_t \cdot \alpha_{t'} \right) \cdot z_{0t'1}(n_s, \alpha_{t'}) \cdot r_t \, d\alpha_{t'}$$

$$Z_{2t'2}(n_s) := \frac{-2}{L_t(n_s)^2} \cdot \int_{r_t}^{-r_t} \left(l_t + L(n_s) + r_t \cdot \frac{\pi}{2} + r_t - x' \right) \cdot z_{0t'2}(n_s, x') \, dx'$$

$$Z_{2t'3}(n_s) := \frac{2}{L_t(n_s)^2} \cdot \int_0^{\frac{\pi}{2}} \left(l_t + L(n_s) + r_t \cdot \frac{\pi}{2} + 2 \cdot r_t + r_t \cdot \beta_{t'}\right) \cdot z_{0t'3}(n_s, \beta_{t'}) \cdot r_t \, d\beta_{t'}$$

$$Z_{2t'}(n_s) := Z_{2t'1}(n_s) + Z_{2t'2}(n_s) + Z_{2t'3}(n_s)$$

$$Z_{2ACD}(n_s) := Z_{2t}(n_s) + Z_{2s}(n_s) + Z_{2t'}(n_s)$$

Comparaison des centres de masses du second ordre

$n_s := 8, 8.02 .. 12$